Golden Connection

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Let's Prove:

$$\phi = \frac{ie^{\frac{\pi i}{10}}}{e^{\frac{\pi i}{5}} - 1} \tag{1}$$

Proof:

0.1 Euler's Formula:

$$e^{xi} = cos(x) + isin(x) \tag{2}$$

0.2 Relation of ϕ and sin(x) and cos(x):

$$sin\left(\frac{\pi}{10}\right) = \frac{1}{2\phi} \tag{3}$$

$$cos\left(\frac{\pi}{10}\right) = \frac{\sqrt{4\phi^2 - 1}}{2\phi} \tag{4}$$

0.3 Combining equations (2), (3) and (4):

$$\begin{aligned} e^{\frac{\pi i}{10}} &= cos\left(\frac{\pi}{10}\right) + i.sin\left(\frac{\pi}{10}\right) \\ &= \frac{\sqrt{4\phi^2 - 1}}{2\phi} + i.\frac{1}{2\phi} \\ &= \frac{\sqrt{4\phi^2 - 1} + i}{2\phi} \\ &= \frac{\sqrt{4(\phi + 1) - 1} + i}{2\phi} \\ &= \frac{\sqrt{4\phi + 3} + i}{2\phi} \end{aligned}$$

$$=> \phi = \frac{i + \sqrt{4\phi + 3}}{2e^{\pi i}}$$

$$=> (2\phi e^{\frac{\pi i}{10}} - i)^2 = 4\phi + 3$$

$$=> 4(\phi + 1)e^{\frac{\pi i}{5}} + i^2 - 4i\phi e^{\frac{\pi i}{10}} = 4\phi + 3$$

$$=> 4\phi e^{\frac{\pi i}{5}} + 4e^{\frac{\pi i}{5}} - 1 - 4i\phi e^{\frac{\pi i}{10}} = 4\phi + 3$$

$$=> 4\phi e^{\frac{\pi i}{5}} + 4e^{\frac{\pi i}{5}} - 4i\phi e^{\frac{\pi i}{10}} - 4\phi = 4$$

$$=> \phi e^{\frac{\pi i}{5}} + e^{\frac{\pi i}{5}} - i\phi e^{\frac{\pi i}{10}} - \phi = 1$$

$$=> \phi(e^{\frac{\pi i}{5}} - ie^{\frac{\pi i}{10}} - 1) = 1 - e^{\frac{\pi i}{5}}$$

$$=> \phi = -\left[\frac{(e^{\frac{\pi i}{5}} - 1)}{e^{\frac{\pi i}{5}} - ie^{\frac{\pi i}{10}} - 1}\right]$$

$$=> \frac{1}{\phi} = -\left[\frac{e^{\frac{\pi i}{5}} - ie^{\frac{\pi i}{10}} - 1}{(e^{\frac{\pi i}{5}} - 1)}\right]$$

$$=> \frac{1}{\phi} = -\left[1 - \frac{ie^{\frac{\pi i}{10}}}{e^{\frac{\pi i}{5}} - 1}\right]$$

$$=> \frac{1}{\phi} = \frac{ie^{\frac{\pi i}{10}}}{e^{\frac{\pi i}{5}} - 1} - 1$$

$$=> \frac{1}{\phi} + 1 = \frac{ie^{\frac{\pi i}{10}}}{e^{\frac{\pi i}{5}} - 1}$$

$$=> \frac{1 + \phi}{\phi} = \frac{ie^{\frac{\pi i}{10}}}{e^{\frac{\pi i}{5}} - 1}$$

$$=> \frac{\phi^2}{\phi} = \frac{ie^{\frac{\pi i}{10}}}{e^{\frac{\pi i}{5}} - 1}$$

$$=> \phi = \frac{ie^{\frac{\pi i}{10}}}{e^{\frac{\pi i}{5}} - 1}$$